Major difference
between true bosons and "proteons"

M. Combescot and O. Betbeder-Matibet

GPS, Université Pierre et Marie Curie and Université Denis Diderot, CNRS,

Campus Boucicaut, 140 rue de Lourmel, 75015 Paris, France

Abstract

We call "proteons" — from the ever-changing greek sea-god $\Pi\rho\omega\tau\epsilon\upsilon\varsigma$ — composite particles made of two fermions. Among them, are the semiconductor excitons, but also various atoms and molecules, like the giant molecules made of two ^{40}K or ^{6}Li atoms which have recently Bose condensed. In addition to their indistinguishability, these composite particles are "ever-changing" in the sense that there is no way to know with which fermions they are precisely made. As direct consequences, (i) the proteons are not true bosons, (ii) the basis made with proteon states is *overcomplete*. In spite of these difficulties, these proteons do have a nice closure relation, unexpected at first, *different from the boson one* and which makes the bosonization procedures used up to now to treat many-body effects between composite bosons, rather questionable, due to possibly incorrect sum rules resulting from it. This closure relation in particular explains, in a neat way, the surprising factor 1/2 between the inverse lifetime and the sum of scattering rates which exists for exact excitons but not for boson excitons, as we have recently shown.

1

The excitons, *i. e.*, the *one* electron-hole (e-h) pair eigenstates of the semiconductor Hamiltonian, are usually considered as bosons. And indeed, various attempts have been devoted to the experimental observation [1-3] of their Bose-Einstein condensation. However, if we are interested in many-body effects, we must be aware that excitons are composite particles, so that they feel each other, not only through Coulomb interactions between carriers, but also through Pauli exclusion between the fermions which compose them. This Pauli way for excitons to interact, which does not need any Coulomb process to exist, is N-body "at once": Indeed, any new exciton feels *all* the excitons present in the sample since it has to be made with fermion states different from the ones already used to make the previous excitons. This "Pauli interaction", which is a direct consequence of the fact that the excitons are not true bosons, induces effects as large as the ones induced by Coulomb interactions. They can even be dominant, as we have recently shown [4], in the case of the semiconductor nonlinear susceptibility $\chi^{(3)}$.

In order to take exactly into account this Pauli way for excitons to interact, we have recently developed a new many-body theory for composite bosons [5-8]. It relies on two scatterings: $\xi^{\mathrm{dir}}_{mnij}$ corresponds to *direct Coulomb* processes between two excitons, the "in" and "out" excitons (i, j) and (m, n) being made with the same pairs, while λ_{mnij} corresponds to a *fermion exchange* between the "in" and "out" excitons, *in the absence of any Coulomb process.*

This new procedure has to be contrasted with the previous approaches in which the exact semiconductor Hamiltonian H is replaced by an effective Hamiltonian H_{eff} written in terms of boson-exciton operators, the composite nature of the original excitons being hidden in the two-body potential of this Hamiltonian, through a scattering "dressed by exchange". Even if the H_{eff} widely used [9] by semiconductor physicists has to be rejected because it is not hermitian — a major failure apparently missed by every one — the concept of an effective bosonic Hamiltonian for excitons is very appealing at first: Indeed, if we manage to replace H by $H_{\mathrm{x}} + V_{\mathrm{xx}}$, we can then treat many-body effects between excitons in a standard way, as all known procedures dealing with interactions rely on a perturbative expansion in the interacting potential. Unfortunately, such a V_{XX} potential does not exist for composite excitons: Indeed, $V_{eh'}$ has to be considered as a part of a potential between two excitons if we see them as made of (e, h) and (e', h'), while the

same $V_{eh'}$ is an "internal" potential if the excitons are made of (e, h') and (e', h). Since the carriers are indistinguishable, there is no way to know !

We have first questioned the concept of bosonized Hamiltonian when we calculated correlation effects between excitons [7]. The major alert however came very recently [10], when we compared the inverse lifetime and the sum of scattering rates of exciton states, calculated with exact excitons using our new many-body theory, and with bosonized excitons using an effective bosonic Hamiltonian: An unexpected additional factor $1/2$ appears for exact excitons, which makes any agreement between the exact results and the ones obtained with boson-excitons, impossible, *whatever the scatterings used in V_{xx} are.*

When we bosonize the excitons, we irretrievably lose their composite nature. The purpose of this letter is to show that it is not enough to repair this loss by the introduction of appropriate X-X scatterings "dressed by exchange". The composite nature of the excitons is quite deep. It also appears through the closure relation between exciton states, which is definitely different from the one of boson-excitons. At first, we did not expect such a closure relation for exact exciton states to exist, because these states are somewhat ticklish : (i) They are non-orthogonal; (ii) worse, they form an overcomplete set. Indeed, for $N = 2$ already, we have (see eq. (7) of ref. [6])

$$\langle v | B_m B_n B_i^\dagger B_j^\dagger | v \rangle = \delta_{mi}\delta_{nj} + \delta_{mj}\delta_{ni} - 2\lambda_{mnij} , \tag{1}$$

where $B_i^\dagger$ is the creation operator of the exact exciton i, (orthogonal states would only have the δ terms), while the overcompleteness of these exciton states follows from

$$B_i^\dagger B_j^\dagger = -\sum_{mn} \lambda_{mnij} B_m^\dagger B_n^\dagger , \tag{2}$$

(see eq. (5) of ref. [6]) which results from the two ways to make two excitons out of two e-h pairs. This overcompleteness is actually trivial to see: If $\mathcal{N}$ is the number of electron (or hole) states in the sample, the possible number of e-h pairs is $\mathcal{N}^2$, just as the possible number of i excitons, since both, the free pairs and the excitons, are eigenstates of the same one-pair Hamiltonian. If we now turn to two-free-pair states, their number is $\mathcal{N}^2(\mathcal{N}-1)^2$, due to Pauli exclusion, while the possible number of two-exciton states (i, j) is $\mathcal{N}^4$: For large $\mathcal{N}$, there are essentially $2\mathcal{N}^3$ exciton states in excess !

It is clear that, if a closure relation, different from the boson-exciton one, has to exist for exact-exciton states, to bosonize the excitons must have dramatic consequences:

3

Indeed, all sum rules which rely on this closure relation, will be different for exact and boson excitons, making the agreement of the corresponding quantities impossible.

In this letter, we are going to speak in terms of excitons, because they are the familiar bosons for us. In addition, their Hamiltonian is nicely simple, being the one of fermions in Coulomb interaction, so that they are under complete control. We however wish to stress that the question raised here, has a much wider interest. Indeed, particles composed of fermions which look like bosons, exist in many other fieds than semiconductor physics, and attempts to bosonize them always have had the same purpose: to possibly handle their interactions, in the absence of a many-body theory which can take care of their composite nature properly. Among composite bosons of current interest, we can cite the molecules made of two ^{40}K or ^{6}Li atoms for which a Bose-Einstein condensation has recently been observed [11-14]. Compared to ^{7}Li atoms for which a BEC was first observed [15-16], these molecules are quite large so that their interactions, in particular through "Pauli scatterings", are expected to play a role as important as for excitons.

In order to give a wider audience to what is here said on excitons, it thus appeared to us of interest to introduce a new particle concept: *the proteon.* Like the ever-changing greek sea-god $\Pi\rho\omega\tau\epsilon\upsilon\varsigma$ (Proteus), there is no way to know with which particular fermions these particles are made. Being moreover indistinguishable, these proteons in fact correspond to a new type of quantum particles, far more tricky than the fermions or the (true) bosons: Indeed, due to their ever-changing character, there is no way to write down an *interacting potential between them*, so that completely new procedures have to be constructed in order to treat their many-body effects properly. This is the goal of the set of works we are presently doing.

Boson-excitons versus exact-excitons

If the excitons were true bosons, their creation operators, $\bar{B}_i^\dagger$, would obey the commutation rule $[\bar{B}_j, \bar{B}_i^\dagger] = \delta_{ij}$. From it, we can show that the N-boson-exciton states $\bar{B}_{i_1}^\dagger \cdots \bar{B}_{i_N}^\dagger |v\rangle$ form an *orthogonal* basis, with a closure relation given by

$$I = \frac{1}{N!} \sum_{i_1 \cdots i_N} \bar{B}_{i_1}^\dagger \cdots \bar{B}_{i_N}^\dagger |v\rangle \langle v| \bar{B}_{i_N} \cdots \bar{B}_{i_1} \ , \tag{3}$$

as easy to check by inserting this identity into $\langle v| \bar{B}_{j_N} \cdots \bar{B}_{j1} |\bar{\psi}\rangle$, where $|\bar{\psi}\rangle$ is an arbitrary N-boson state. Using eq. (3), we can then expand any $|\bar{\psi}\rangle$ on the N-boson-exciton states

4

in an unique way, due to the orthogonality of these $\bar{B}_{i_1}^\dagger \cdots \bar{B}_{i_N}^\dagger |v\rangle$ states.

For exact-excitons, the situation is far more subtle:

(i) Using

$$[D_{mi}, B_j^\dagger] = 2\sum_n \lambda_{mnij}\, B_n^\dagger \ , \tag{4}$$

where $D_{mi} = \delta_{mi} - [B_m, B_i^\dagger]$ is the "deviation-from-boson operator", it is formally possible to calculate the scalar product of N-exciton states $B_{i_1}^\dagger \cdots B_{i_N}^\dagger |v\rangle$: For $N = 2$, it is given by eq. (1) — which immediately follows from eq. (4). For large N, these scalar products are far more tricky. When all the excitons are in the same ground state 0, we have already shown [5] that $\langle v|B_0^N B_0^{\dagger N}|v\rangle$ reads $N!F_N$. While F_N reduces to 1 for boson-excitons, for exact excitons and N large, it is a complicated function of N and $\eta = Na_X^3/\mathcal{V}$, which, in the small η limit, behaves [17] as $\exp -(33\pi N\eta/4)$ in 3D. Although η has to be small for the excitons to exist, $N\eta$ can be much larger than 1 for large samples, making F_N extremely small. In previous works, we have also calculated the scalar products of N-exciton states with one or two excitons possibly outside the ground state [18,19]. Their expressions do not reduce to products of Kronecker symbols as for orthogonal states, but read in terms of N, F_{N-p} and the λ_{mnij}'s in a very complicated way.

(ii) In addition, the decomposition of a N-electron-hole-pair state on these N-exciton states is *a priori* not unique: Due to eq. (2), we for example have,

$$B_0^{\dagger N}|v\rangle = -\sum_{mn} \lambda_{mn00} B_m^\dagger B_n^\dagger B_0^{\dagger N-2}|v\rangle \ . \tag{5}$$

In spite of these obvious difficulties, the N-exciton states are physically appealing in the low density limit because, in this limit, the e-h pairs are known to form excitons. If we for example consider the N-pair ground state, its representation in terms of free electrons and free holes has to contain a lot of e-h states with essentially equal weight, while its representation in terms of excitons should be mainly made of $B_0^{\dagger N}|v\rangle$.

However, in order for these exciton states to be of practical use, we must find a simple procedure to express any N-pair state $|\psi\rangle$ on these N-exciton states. Although the idea was far from our mind at first, a closure relation, as simple as eq. (3), in fact exists for exact-excitons.

Closure relation for exciton states

Let us first consider a two-e-h–pair state $|\psi\rangle$. Its expansion on two-exciton states must read $|\psi\rangle = \sum_{i,j} \psi_{ij} B_i^\dagger B_j^\dagger |v\rangle$. The ψ_{ij}'s *a priori* depend on $|\psi\rangle$ and on the indices (i,j) of the $B_i^\dagger B_j^\dagger |v\rangle$ state of interest in the expansion. The simplest-minded idea is to try $\psi_{ij} = \langle v|B_i B_j|\psi\rangle$ with possibly an additional (i,j) dependent prefactor. At first, we could think of a prefactor being a constant α divided by $\langle v|B_i B_j B_i^\dagger B_j^\dagger |v\rangle$, in order for the two-exciton state $B_i^\dagger B_j^\dagger |v\rangle$ to appear in a normalized form in the expansion. It turns out that this is not correct: The prefactor of $\langle v|B_i B_j|\psi\rangle$ is $(1/4)$ for all (i,j), the correct expansion being

$$|\psi\rangle = \frac{1}{4} \sum_{ij} B_i^\dagger B_j^\dagger |v\rangle \langle v|B_i B_j|\psi\rangle \ , \tag{6}$$

as easy to check by calculating the scalar product of this $|\psi\rangle$ with $\langle v|B_m B_n$, using eqs. (1,2). From the equivalent of eq. (1) for three-exciton states, it is possible to show that the expansion of three-pair states is quite similar to eq. (6), with a prefactor $(1/36)$ instead of $(1/4)$.

This led us to think that the closure relation for N-exciton states should read

$$I = \frac{1}{(N!)^2} \sum_{i_1 \cdots i_N} B_{i_1}^\dagger \cdots B_{i_N}^\dagger |v\rangle \langle v|B_{i_N} \cdots B_{i_1} \ . \tag{7}$$

In order to prove it, the procedure used for $N = (2,3)$ is however inappropriate because we do not know how to write the scalar product of N-exciton states in a compact form.

Another way to show it, is to expand the exciton operators in terms of electrons and holes,

$$B_i^\dagger = \sum_{\mathbf{k}_e, \mathbf{k}_h} \langle \mathbf{k}_e, \mathbf{k}_h|\phi_i\rangle \, a_{\mathbf{k}_e}^\dagger b_{\mathbf{k}_h}^\dagger \ , \tag{8}$$

where $|\phi_i\rangle$ is the i exciton wave function, and to use this expansion in the r.h.s. of eq. (7). This leads to

$$\frac{1}{(N!)^2} \sum_{\{i_n\}} B_{i_1}^\dagger \cdots B_{i_N}^\dagger |v\rangle \langle v|B_{i_N} \cdots B_{i_1} = \frac{1}{(N!)^2} \sum_{\{\mathbf{k}_n\},\{\mathbf{k}_n'\},\{\mathbf{p}_n\},\{\mathbf{p}_n'\}} a_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_1'}^\dagger \cdots a_{\mathbf{k}_N}^\dagger b_{\mathbf{k}_N'}^\dagger |v\rangle$$

$$\langle v|b_{\mathbf{p}_N'} a_{\mathbf{p}_N} \cdots b_{\mathbf{p}_1'} a_{\mathbf{p}_1} \prod_{n=1}^{N} \left(\sum_{i_n} \langle \mathbf{k}_n, \mathbf{k}_n'|\phi_{i_n}\rangle \langle \phi_{i_n}|\mathbf{p}_n', \mathbf{p}_n\rangle \right) \ . \tag{9}$$

The summation over i_n can be done through the closure relation which exists between *one*-exciton states. It gives $\delta_{\mathbf{k}_n,\mathbf{p}_n} \delta_{\mathbf{k}_n',\mathbf{p}_n'}$, so that the r.h.s. of eq. (9) reduces to

$$\frac{1}{(N!)^2} \sum_{\{\mathbf{k}_n\},\{\mathbf{k}_n'\}} a_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_1'}^\dagger \cdots a_{\mathbf{k}_N}^\dagger b_{\mathbf{k}_N'}^\dagger |v\rangle \langle v|b_{\mathbf{k}_N'} a_{\mathbf{k}_N} \cdots b_{\mathbf{k}_1'} a_{\mathbf{k}_1} \ , \tag{10}$$

which is nothing but I, due to the closure relation which exists between electron states, namely

$$I = \frac{1}{N!} \sum_{\mathbf{k}_n} a^\dagger_{\mathbf{k}_1} \cdots a^\dagger_{\mathbf{k}_N} |v\rangle\langle v| a_{\mathbf{k}_N} \cdots a_{\mathbf{k}_1} , \tag{11}$$

and the similar one between hole states.

This derivation shows in a transparent way that the prefactor $(1/N!)^2$, instead of $(1/N!)$, found in the closure relation of exact excitons is nothing but the direct signature of the composite nature of these excitons. When they are bosonized, this composite nature is lost by construction and the closure relation for N boson-excitons has the usual $(1/N!)$ prefactor, characteristic of elementary quantum particles.

Link between the lifetime and the sum of scattering rates of excitons

In the light of these different closure relations, let us reconsider the link between the lifetime and the sum of scattering rates of exact and boson excitons we recently found [10]. In order to point out the physical origin of the striking factor $1/2$, which appears for exact excitons, in the easiest way, we will consider $N = 2$ excitons only.

The closure relation for two exact excitons, given in eq. (6), also reads

$$I = \frac{1}{4}\left[\sum_i (2 - 2\lambda_{iiii})|\phi_{ii}\rangle\langle\phi_{ii}| + \sum_{i\neq j}(1 - 2\lambda_{ijij})|\phi_{ij}\rangle\langle\phi_{ij}|\right] , \tag{12}$$

where, according to eq. (1), the $|\phi_{ij}\rangle$'s, defined as $|\phi_{ij}\rangle = (1 + \delta_{ij} - 2\lambda_{ijij})^{-1/2}B_i^\dagger B_j^\dagger|v\rangle$, are the normalized two-exciton states.

As the standard Fermi golden rule cannot be used to obtain the lifetime and scattering rates of exact excitons, because there is no way to write down an interacting potential *between* excitons, we have been forced to generate unconventional expressions of these quantities in which only enters the Hamiltonian H. Of course, these expressions can also be used for boson excitons. This led us to look for the time evolution of an initial state $|\psi_{t=0}\rangle$ as

$$|\psi_t\rangle = e^{-i(H - \langle H\rangle)t}|\psi_{t=0}\rangle = |\psi_{t=0}\rangle + |\tilde{\psi}_t\rangle , \tag{13}$$

where $\langle H\rangle = \langle\psi_{t=0}|H|\psi_{t=0}\rangle$ just adds an irrelevant phase factor to the standard expression of $|\psi_t\rangle$. For $|\psi_{t=0}\rangle = |\phi_{00}\rangle$, the state $|\tilde{\psi}_t\rangle$, precisely given by

$$|\tilde{\psi}_t\rangle = F_t(H - \langle H\rangle)[H - \langle H\rangle]|\phi_{00}\rangle , \tag{14}$$

7

where $F_t(E) = (e^{-iEt} - 1)/E$, is nothing but the initial state change induced by its time evolution due to the exciton-exciton Coulomb scatterings ξ^{dir}_{mnij}: We do in particular have $(H - 2E_0)B^{\dagger 2}_0|v\rangle = \sum_{mn} \xi^{\text{dir}}_{mn00} B^{\dagger}_m B^{\dagger}_n|v\rangle$, where E_0 is the energy of the 0 exciton, $(H - E_0)B^{\dagger}_0|v\rangle = 0$ (see eq. (6) of ref. [6]). In the absence of these scatterings, $H|\phi_{00}\rangle$ reduces to $2E_0|\phi_{00}\rangle$, so that $(H - \langle H\rangle)|\phi_{00}\rangle$ reduces to zero, as well as $|\tilde{\psi}_t\rangle$. The state change $|\tilde{\psi}_t\rangle$ in fact vanishes linearly with the exciton-exciton Coulomb scatterings, as $H|\phi_{00}\rangle$ is linear in Coulomb interaction.

From $|\psi_t\rangle$, we can get the initial state lifetime through

$$e^{-t/\tau_0} = |\langle\psi_{t=0}|\psi_t\rangle|^2 = 1 + [\langle\phi_{00}|\tilde{\psi}_t\rangle + c.c] + \left|\langle\phi_{00}|\tilde{\psi}_t\rangle\right|^2 . \tag{15}$$

Although the dominant term of $|\tilde{\psi}_t\rangle$ is linear in exciton-exciton Coulomb scatterings, the dominant term of its scalar product with $|\phi_{00}\rangle$ is quadratic only. Indeed, if we replace $\langle\phi_{00}|F_t(H - \langle H\rangle)$ by its zero order contribution, namely $(-it)\langle\phi_{00}|$, we see, from eq. (14), that $\langle\phi_{00}|\tilde{\psi}_t\rangle$ cancels. We then note that, as $\langle\psi_t|\psi_t\rangle = 1$, the real part of $\langle\phi_{00}|\tilde{\psi}_t\rangle$ is just $(-1/2)\langle\tilde{\psi}_t|\tilde{\psi}_t\rangle$, which is indeed quadratic in scatterings. So that, to second order in Coulomb interaction, the lifetime is simply given by

$$\frac{t}{\tau_0} \simeq \langle\tilde{\psi}_t|\tilde{\psi}_t\rangle , \tag{16}$$

since the last term of eq. (15) is fourth order.

If we now inject the closure relation between exact excitons given in eq. (12) into this equation (16), we find

$$\frac{t}{\tau_0} \simeq \frac{1}{4}\left[(2 - 2\lambda_{0000})\left|\langle\phi_{00}|\tilde{\psi}_t\rangle\right|^2 + \sum_{i\neq 0}(2 - 2\lambda_{iiii})\frac{t}{T_{ii}} + \sum_{i\neq j}(1 - 2\lambda_{ijij})\frac{t}{T_{ij}}\right] , \tag{17}$$

where the T^{-1}_{ij}'s, defined as $t/T_{ij} = \left|\langle\phi_{ij}|\tilde{\psi}_t\rangle\right|^2$, are the scattering rates towards other exciton states induced by the time evolution of $|\psi_{t=0}\rangle$. Note that this definition, instead of $|\langle\phi_{ij}|\psi_t\rangle|^2$, insures these scattering rates to be really linked to the state change induced by the initial state time evolution, even if the final states are not orthogonal to the initial state.

As physically expected, and possibly checked from microscopic calculations [10], these scattering rates contain an energy conservation which imposes the (i, j) states to be close in energy to $(0, 0)$; so that the states reached by the time evolution of $|\phi_{00}\rangle$, with

8

$0 = (\nu_0, \mathbf{0})$, must have the same relative motion index ν_0. Due to momentum conservation in the scattering processes, these states in fact are $i = (\nu_0, \mathbf{q})$ and $j = (\nu_0, -\mathbf{q})$. Consequently, $T_{ii}^{-1} = 0$ for $i \neq 0$. As the first term of eq. (17) is of the order of the last term of eq. (15) we have dropped, we end with

$$\frac{1}{\tau_0} \simeq \frac{1}{2} \sum_{(i,j)couples} \frac{1}{T_{ij}} \, , \tag{18}$$

in the large sample limit, since we have shown that, for excitons having a bound relative motion, the λ_{mnij}'s are of the order of the exciton volume divided by the sample volume, making these λ's negligible in front of 1.

For boson-excitons, the calculation is exactly the same, except that all the λ_{mnij}'s are equal to zero, while the $1/4$ prefactor of the closure relation (12) is replaced by $1/2$. This change leads to drop the $1/2$ in front of the sum in eq. (18). This shows in a quite direct way that the relations between the lifetime and the sum of scattering rates of exact and boson excitons have to differ by a factor $1/2$, in agreement with the microscopic calculations of τ_0 and the T_{ij}'s we have recently done [10].

In the case of N excitons, the closure relation for exact excitons contains an additional prefactor $(1/N!)$, instead of $(1/2!)$ as for $N = 2$. It is however clear that this $(1/N!)$ cannot barely appear in front of the sum of scattering rates of N exact excitons, otherwise the lifetime of these N excitons would tend to zero in the large N limit, which is physically unreasonable. And indeed, our microscopic calculation of τ_0 and the T_{ij}'s shows that the same factor $(1/2)$ exists between τ_0^{-1} and the sum of T_{ij}^{-1}'s, as for $N = 2$. The proof that these various factors N do ultimately disappear, which is not at all trivial, is beyond the scope of this paper. It is somehow related to the bosonic enhancement factors we recently found for excitons embedded in a sea of excitons [19].

As a conclusion, this letter allows to clearly show that the unexpected factor $1/2$ we recently found between the lifetime and the sum of scattering rates of exact excitons, physically comes from the composite nature of these excitons: They are deeply made of two fermions and there is no way to get rid of this fact. This composite nature makes the exciton state set overcomplete, with a closure relation different from the one of elementary particles, so that all sum rules deduced from it have to appear differently. Similar results are *a priori* expected for composite bosons in other fields than semiconductor physics. Our letter neatly shows that these composite bosons should not be reduced to true bosons

with an interaction dressed by exchange, as commonly done: They actually form a new class of quantum particles, the "proteons", their many-body effects having to be handled through the new theory for composite bosons we have recently developed, if we want to fully trust them.

REFERENCES

(1) D. Hulin, A. Mysyrowicz and C. Benoit à la Guillaume, *Phys. Rev. Lett.* **45**, 1971 (1980).

(2) D.W. Snoke, J.P. Wolfe and A. Mysyrowicz, *Phys. Rev. Lett.* **64**, 2543 (1990).

(3) L.V. Butov, A.C. Gossard and D.S. Chemla, *Nature* **418**, 751 (2002).

(4) M. Combescot, O. Betbeder-Matibet, K. Cho and H. Ajiki, *Cond-mat*/0311387.

(5) M. Combescot and C. Tanguy, *Europhys. Lett.* **55**, 390 (2001).

(6) M. Combescot and O. Betbeder-Matibet, *Europhys. Lett.* **58**, 87 (2002).

(7) M. Combescot and O. Betbeder-Matibet, *Europhys. Lett.* **59**, 579 (2002).

(8) O. Betbeder-Matibet and M. Combescot, *Eur. Phys. J. B* **27**, 505 (2002).

(9) See for example H. Haug and S. Schmitt-Rink, *Prog. Quantum Electron.* **9**, 3 (1984).

(10) M. Combescot and O. Betbeder-Matibet, *Cond-mat*/0402087, to be published in *Phys. Rev. Lett.* (2004).

(11) M. Greiner, C.A. Regal and D.S. Jin, *Nature* **426**, 537 (2003).

(12) S. Jochim *et al., Science* **302**, 2102 (2003).

(13) M.W. Zwierlein *et al., Phys. Rev. Lett.* **91**, 250401 (2003).

(14) T. Bourdel *et al., Cond-mat*/0403091.

(15) M.H. Anderson *et al., Science* **269**, 198 (1995).

(16) K.B. Davis *et al., Phys. Rev. Lett.* **75**, 3969 (1995).

(17) M. Combescot, X. Leyronas and C. Tanguy, *Eur. Phys. J. B* **31**, 17 (2003).

(18) O. Betbeder-Matibet and M. Combescot, *Eur. Phys. J. B* **31**, 517 (2003)

(19) M. Combescot and O. Betbeder-Matibet, *Cond-mat*/0403073.